Number Conversions

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INTRODUCTION

Number System- A number system defines a set of values to represent quantity. We talk about the number of people attending a class, the number of modules taken by each student and use numbers to represent grade.

Number System can be categorized in two systems:-

(a) Non-Positional Number System
(b) Positional Number System

Non-Positional Number System- In ancient times, people used to count on fingers, when the fingers became insufficient for counting, stones, pebbles or sticks were used to indicate values. But it was very difficult to perform arithmetic with such a number system as there is no symbol for zero.

Positional Number System- In this system the value of each digit is defined not by the symbol but also by the symbol position. Positional Number System is used to perform arithmetic. Existing Positional number system is decimal number system. Apart from the decimal number system, there are binary number system, octal number system and hexadecimal number system.

Base (Radix)- In the number system the base or radix tells the number of symbols used in the system. In the earlier days, different civilisations were using different radixes. The Egyptian used the radix 2, the Babylonians used the radix 60 and Mayans used 18 and 20.
The base of a number system is indicated by a subscript (decimal number) and this will be followed by the value of the number. For example $(952)_{10}$, $(456)_8$, $(314)_{16}$

Number System that are used by the computers-
Decimal System
Binary System
Octal System
Hexadecimal System

Decimal System- The decimal system is the system which we use in everyday counting. The number system includes the ten digits from 0 through 9. These digits are recognized as the symbols of the decimal system. Each digit in a base ten number represents units ten times the units of the digit to its right.
For example-
$9542= 9000 + 500 + 40 +2= (9 \times 10^3) + (5 \times 10^2) + (4 \times 10) + (2\times 10^0)$

Binary System - Computers do not use the decimal system for counting and arithmetic. Their CPU and memory are made up of millions of tiny switches that can be either in ON and OFF states. **0** represents OFF and **1** represents ON. In this way we use binary system.

Binary system has two numbers 0 and 1. Binary system has base 2 therefore the weight of n^{th} bit of the number from Right Hand Side is n^{th} bit × 2^{n-1}.

Octal System- The octal system is commonly used with computers. The octal number system with its 8 digit 0,1,2,3,4,5,6, and 7 has base 8. The octal system uses a power of 8 to determine the digit of a number's position.

Hexadecimal System- Hexadecimal is another number system that works exactly like the decimal, binary and octal number systems, except that the base is 16. Each hexadecimal represents a power of 16. The system uses 0 to 9 numbers and A to F characters to represent 10 to 15 respectively.

Conversions- Any number in one number system can be converted into any other number system. There are the various methods that are used in converting numbers from one base to another.

Conversions of Decimal to Binary- The method that is used for converting of decimals into binary is known as the remainder method. We use the following steps in getting the binary number-

(a) Divide the decimal number by 2.
(b) Write the remainder (which is either 0 or 1) at the right most position.
(c) Repeat the process of dividing by 2 until the quotient is 0 and keep writing the remainder after each step of division.
(d) Write the remainders in reverse order.

Example- Convert $(45)_{10}$ into binary number system.

2	45	**Remainder**
2	22	1
2	11	0
2	5	1
2	2	1
2	1	0
	0	1

Thus $(45)_{10} = (101101)_2$

Note- In every number system-

(a) The first bit from the right is referred as LSB (Least Significant Bit)

(b) The first bit from the left is referred as MSB (Most Significant Bit)

Conversions of Decimal Fractions to Binary Fractions- For converting decimal fractions into binary fractions, we use multiplication. Instead of looking for a remainder we look for an integer. The following steps are used in getting the binary fractions-

(a) Multiply the decimal fraction by 2.

(b) If a non-zero integer is generated, record the non-zero integer otherwise record 0.

(c) Remove the non-zero integer and repeat the above steps till the fraction value becomes 0.

(d) Write down the number according to the occurrence.

Example- Find the binary equivalent of $(0.75)_{10}$.

	Number (to be recorded)
$0.75 \times 2 = 1.50$	1
$0.50 \times 2 = 1.00$	1

Thus $(0.75)_{10} = (0.11)_2$.

Moreover, we can write $(45.75)_{10} = (101101.11)_2$.

Remark- If the conversion is not ended and still continuing; we write the approximation in 16 bits.

Example- Find the binary equivalent of $(0.9)_{10}$.

	Number (to be recorded)
$0.9 \times 2 = 1.8$	1
$0.8 \times 2 = 1.6$	1
$0.6 \times 2 = 1.2$	1
$0.2 \times 2 = 0.4$	0
$0.4 \times 2 = 0.8$	0
$0.8 \times 2 = 1.6$	1
$0.6 \times 2 = 1.2$	1
$0.2 \times 2 = 0.4$	0
$0.4 \times 2 = 0.8$	0
$0.8 \times 2 = 1.6$	1
$0.6 \times 2 = 1.2$	1
$0.2 \times 2 = 0.4$	0
$0.4 \times 2 = 0.8$	0
$0.8 \times 2 = 1.6$	1
$0.6 \times 2 = 1.2$	1
$0.2 \times 2 = 0.4$	0
$0.4 \times 2 = 0.8$	0
$0.8 \times 2 = 1.6$	1

Thus $(0.9)_{10} = (0.111001100110011001)_2$.

Conversion of Decimal to Octal- In converting decimal to octal, we follow the same process of converting decimal to binary. Instead of dividing the number by 2, we divide the number by 8.

Example- Convert $(45)_{10}$ into octal number system.

		Remainder
8	45	
8	5	5
8	0	5

Thus $(45)_{10}= (55)_8$.

Conversions of Decimal Fractions to Octal Fractions –We follow the same steps of conversions of decimal fractions to binary fractions. Here we multiply the fraction by 8 instead of 2.

Example- Find the octal equivalent of $(0.75)_{10}$.

	Number (to be recorded)
$0.75 \times 8 = 6.00$	6

Thus $(0.75)_{10}= (0.6)_8$.

And $(45.75)_{10}= (55.6)_8$.

Conversion of Decimal to Hexadecimal – We divide by 16 instead of 2 or 8. If the remainder is in between 10 to 16, then the number is represented by A to F respectively.

Example- Convert $(45)_{10}$ into hexadecimal.

		Remainder
16	45	
16	2	D
16	0	2

Thus $(45)_{10}= (2D)_{16}$.

Conversions of Decimal Fractions to Hexadecimal Fractions – Here we multiply the fraction by 16 instead of 2 or 8. If the non-zero integer is in between 10 to 16, then the number is represented by A to F respectively.

Example- Find the hexadecimal equivalent of $(0.75)_{10}$.

	Number (to be recorded)
$0.75 \times 16 = 12.00$	C (12 = C)

Thus $(0.75)_{10} = (0.C)_{16}$.

And $(45.75)_{10} = (2D.C)_{16}$.

Conversions of Binary to Decimal – In converting binary to decimal, we use the following steps-

(a) Write the weight of each bit.
(b) Get the weighted value by multiplying the weighted position with the respective bit.
(c) Add all the weighted value to get the decimal number.

Example- Convert $(101101)_2$ into decimal number system.

Binary Number	1	0	1	1	0	1
Wt. of each bit	2^5	2^4	2^3	2^2	2^1	2^0
Weighted Value	1×2^5	0×2^4	1×2^3	1×2^2	0×2	1×2^0
Solved Multiplication	32	0	8	4	0	1

Thus $(101101)_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2 + 1 \times 2^0$.
$= 32 + 0 + 8 + 4 + 0 + 1$
$= 45$

Conversions of Binary Fractions to Decimal Fractions – The conversions of binary fractions to the decimal fractions is similar to conversion of binary numbers to decimal numbers. Here, instead of a decimal point we have a binary point. The exponential expressions (or weight of the bits) of each fractional placeholder is 2^{-1}, 2^{-2}………

Example- Convert $(101101.11)_2$ into decimal number system.

Binary Number	1	0	1	1	0	1	1	1
Wt. of each bit	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
Weighted Value	1×2^5	0×2^4	1×2^3	1×2^2	0×2	1×2^0	1×2^{-1}	1×2^{-2}
Solved Multiplication	32	0	8	4	0	1	0.5	0.25

Thus $(101101.11)_2 = 32 + 0 + 8 + 4 + 0 + 1 + 0.5 + 0.25 = 45.75$

Conversions of Binary to Octal- We use the following steps in converting binary to octal-

(a) Break the number into 3-bit sections starting from LSB to MSB.
(b) If we do not have sufficient bits in grouping of 3-bits, we add zeros to the left of MSB so that all the groups have proper 3-bit number.
(c) Write the 3-bit binary number to its octal equivalent.

Example- Convert $(101101)_2$ into octal.

Binary Number	101	101
Octal Number	5	5

Thus $(101101)_2 = (55)_8$.

Example- Convert $(1101101)_2$ into octal.

Binary Number	001	101	101
Octal Number	1	5	5

Thus $(1101101)_2 = (155)_8$.

Conversions of Binary Fractions to Octal Fractions- We use the following steps in converting binary fractions to octal fractions-

(d) Break the fraction into 3-bit sections starting from MSB to LSB.
(e) In order to get a complete grouping of 3 bits, we add trailing zeros in LSB.
(f) Write the 3-bit binary number to its octal equivalent.

Example- Convert $(101101.11)_2$ into octal.

Binary Number	101	101	110
Octal Number	5	5	6

Thus $(101101)_2 = (55.6)_8$.

Conversions of Binary to Hexadecimal- We convert binary to hexadecimal in the similar manner as we have converted binary to octal. The only difference is that here, we form the group of 4 –bits.

Example- Convert $(101101)_2$ into hexadecimal.

Binary Number	0010	1101
Decimal Number	2	13
Hexadecimal Number	2	D

Thus $(101101)_2 = (2D)_{16}$.

Conversions of Binary Fractions to Hexadecimal Fractions - We convert binary fractions to hexadecimal fractions in the similar manner as we have converted binary fractions to octal fractions. The only difference is that here we form the group of 4 – bits.

Example- Convert $(101101.11)_2$ into hexadecimal.

Binary Number	0010	1101	1100
Decimal Number	2	13	12
Hexadecimal Number	2	D	C

Thus $(101101.11)_2 = (2D.C)_{16}$.

Conversions of Octal to Decimal- We follow the same steps of conversion of binary to decimal. The only difference is that here weight of n^{th} bit is 8^{n-1} instead of 2^{n-1}.

Example- Convert $(55)_8$ into decimal number system.

Octal Number	5	5
Wt. of each bit	8^1	8^0
Weighted Value	5×8	5×8^0
Solved Multiplication	40	5

Thus $(55)_8 = 40 + 5$.
$= 45$

Conversions of Octal Fractions to Decimal Fractions- The weight of the bit of the fraction placeholder is 8^{-1}, 8^{-2}………. We follow the same steps of conversion of binary fractions to decimal fractions.

Example- Convert $(55.6)_8$ into decimal number system.

Octal Number	5	5	6
Wt. of each bit	8^1	8^0	8^{-1}
Weighted Value	5×8	5×8^0	6×8^{-1}
Solved Multiplication	40	5	0.75

Thus $(55.6)_8$= 40 + 5 + 0.75 = 45.75

Conversions of Octal to Binary- We use the following steps in converting octal to binary-

(a) Convert each octal digit into 3-bit binary equivalent.
(b) Combine the 3-bit section by removing the spaces to get the binary number.

Example- Convert $(55)_8$ into binary.

Octal Number	5	5
Binary Number	101	101

Thus $(55)_8 = (101101)_2$.

Example- Convert $(456)_8$ into binary.

Octal Number	4	5	6
Binary Number	100	101	110

Thus $(456)_8 = (100101110)_2$.

Conversions of Octal Fractions to Binary Fractions- We follow the same steps of conversion of octal to binary.

Example- Convert $(55.6)_8$ into binary.

Octal Number	5	5	6
Binary Number	101	101	110

Thus $(55.6)_8 = (101101.11)_2$.

Conversions of Octal to Hexadecimal- The conversion involves the following steps-

(a) Convert each octal digit to 3 –bit binary form.
(b) Combine all the 3-bit binary numbers.
(c) Group them in 4-bit binary form by starting from MSB to LSB.
(d) Convert these 4-bit blocks into their hexadecimal symbols.

Example- Convert $(55)_8$ into hexadecimal.

Octal Number	5	5
Binary Number	101	101

Combining the 3-bit binary block, we have 101101.
Grouping them in 4 bit binary form-

Binary Number	0010	1101
Hexadecimal Symbol	2	D

Thus $(55)_8 = (2D)_{16}$.

Conversions of Octal Fractions to Hexadecimal Fractions- The method of conversion is based on the same procedure that we have discussed in conversions of octal to hexadecimal.

Example- Convert $(55.6)_8$ into hexadecimal.

Octal Number	5	5	6
Binary Number	101	101	110

Combining the 3-bit binary block, we have 101101.110.
Grouping them in 4 bit binary form-

Binary Number	0010	1101	1100
Hexadecimal Symbol	2	D	C

Thus $(55)_8 = (2D.C)_{16}$.

Conversions of Hexadecimal to Decimal- We do the conversion of hexadecimal to decimal as we have done the conversion of binary to decimal. Here weight of n^{th} bit is 16^{n-1} instead of 2^{n-1}.

Example- Convert $(2D)_{16}$ into decimal.

Hexadecimal Number	2	D(=13)
Wt. of each bit	16^1	16^0
Weighted Value	2×16	13×16^0
Solved Multiplication	32	13

Thus $(2D)_{16} = 32 + 13 = 45$.

Conversions of Hexadecimal Fractions to Decimal Fractions- We do the conversion of hexadecimal fractions to decimal fractions in the similar manner as we have done the conversion of binary fractions to decimal fractions. Here weight of bit is 16^{-1}, 16^{-2}…….

Example- Convert $(2D.C)_{16}$ into decimal.

Hexadecimal Number	2	D(=13)	C(=12)
Wt. of each bit	16^1	16^0	16^{-1}
Weighted Value	2×16	13×16^0	13×16^{-1}
Solved Multiplication	32	13	0.75

Thus $(2D.C)_{16} = 32 + 13 + 0.75 = 45.75$.

Conversions of Hexadecimal to Binary-We use the following steps-

(a) Convert each hexadecimal digit to its 4-bit binary equivalent.
(b) Combine all the binary numbers.

Example- Convert $(2D)_{16}$ into binary.

Hexadecimal Number	2	D(=13)
Binary Number	0010	1101

Thus $(2D)_{16} = (00101101)_2 = (101101)_2$.

Conversions of Hexadecimal Fractions to Binary Fractions -We use the same steps of hexadecimal to binary conversion.

Example- Convert $(2D.C)_{16}$ into binary.

Hexadecimal Number	2	D(=13)	C(=12)
Binary Number	0010	1101	1100

Thus $(2D)_{16}$ = $(00101101.1100)_2$= $(101101.11)_2$.

Conversions of Hexadecimal to Octal- We convert each hexadecimal digit in binary. Combine all the binary numbers. Again group them into 3-bit form. Convert the 3-bit block in octal.

Example- Convert $(2D)_{16}$ into octal.

Hexadecimal Number	2	D(=13)
Binary Number	0010	1101

Combining the binary number, we get 00101101=101101
Grouping the binary number in 3-bit

Binary Number	101	101
Octal Number	5	5

Thus $(2D)_{16}$ = $(55)_8$.
Conversions of Hexadecimal Fractions to Octal Fractions – We follow the same steps of hexadecimal to octal conversion.

Example- Convert $(2D.C)_{16}$ into octal.

Hexadecimal Number	2	D(=13)	C(=12)
Binary Number	0010	1101	1100

Combining the binary number, we get 00101101.1100=101101.11
Grouping the binary number in 3-bit

Binary Number	101	101	110
Octal Number	5	5	6

Thus $(2D.C)_{16}$ = $(55.6)_8$.